Math 70 11.4 Applications involving Quadratic equations Lesson 31

Objectives:

1) Solve uniform motion problems
2) Solve a formula for a given variable
3) Use position functions to find
   - position at a given time
   - time at a given position (including "hit the ground")
4) Solve geometry problems, including using the Pythagorean theorem
5) Solve work rate problems

Exam #3.

Math 70 11.4 Applications of Quadratic Equations

Objectives:

1) Solve word problems
   - Work rates
   - Distance, rate, time
   - Geometry

Examples

1) Together, an experienced word processor and an apprentice word processor can create a document in 6 hours. Alone, the experienced one can do the job 2 hours faster than the apprentice can do it working alone. Find the time for each to do the job alone.
2) Beach and Fargo are 400 miles apart. A salesperson travels from Beach to Fargo on Monday , and back to Beach on Tuesday. On Tuesday, she drives 10 mph faster. The total time spent driving was $14\frac{2}{3}$ hours. Find her speed in each direction. Round to the nearest mile per hour, if necessary.
3) Suppose that an open box is to be made from a square sheet of cardboard by cutting 2-inch-by-2-inch squares out of each corner and folding the sides up. The finished box has volume 128 cubic inches. Find he original dimensions of the sheet of cardboard.
4) Students cut diagonally across the lawn instead of walking two sidewalks which form a right angle. The path on the lawn is 20 feet, while they travel x feet and x+2 feet on the two sidewalks. How many feet do they save by cutting across the lawn?
5) A water tank can be filled by the large and small inlet pipes in 3 hours. The large inlet pipe can fill the tank in 2 hours less time than the small inlet pipe. Find the time, to the nearest tenth of an hour, that each pipe takes to fill the tank alone.
6) Bill and his son Billy can clean the house together in 4 hours. Billy takes an hour longer by himself than Bill takes by himself. How long, to the nearest hour, does it take each one to clean the house, if working alone?
7) At the 2007 Grand Prix of Long Beach auto race, Simon Pagenaud posted the fastest lap speed, but Sebastian Bourdais won the race. One lap through the streets of Long Beach is 10,391 feet (1.968 miles) long. Pagenaud's fastest lap speed was 0.55 foot per second faster than Bourdais's fastest lap speed. Traveling at these fastest speeds, Bourdais would have taken 0.25 second longer than Pagenaud to complete a lap.
   a. Find Sebastian Bourdais's fastest lap speed during the race. Round to two decimal places.
   b. Find Simon Pagenaud's fastest lap speed during the race. Round to two decimal places.
   c. Convert each speed to miles per hour. Round to one decimal place.
8) A family drives 500 miles to the Grand Canyon. The return trip was made at an average speed 10 mph faster. The total traveling time was $18\frac{1}{3}$ hours. Find the speed to the Grand Canyon and the return speed.

Math 70

(1)

Together, an experienced word processor and an apprentice word processor can create a document in 6 hours. Alone, the experienced one can do the job 2 hours faster than the apprentice can do it working alone. Find the time for each to do the job alone.

Work rates: fraction done by one alone + fraction done by other alone = fraction done together

together = 6 hrs
experienced = $x-2$ ← faster = less time
apprentice = x

$$\frac{1}{x} + \frac{1}{x-2} = \frac{1}{6}$$

$LCD = 6x(x-2)$

$$\frac{1}{x} \cdot 6x(x-2) + \frac{1}{(x-2)} \cdot 6x(x-2) = \frac{1}{6} \cdot 6x(x-2)$$

order of op: mult before add

$$6(x+2) + 6x = x(x-2)$$

$$6x - 12 + 6x = x^2 - 2x$$

$$12x - 12 = x^2 - 2x$$

$x^2 \Rightarrow$ quadratic
set = 0.

$$0 = x^2 - 14x + 12$$

12: $-1, -12$; $-2, -6$; $-3, -4$ (none add to -14)

does not factor ⇒ quadratic formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{14 \pm \sqrt{148}}{2}$$

$$x \approx 13.08, +.91$$

↑
$x-2$ would be negative so this is extraneous

Math 70

(1) continued

$$X = \frac{14 + \sqrt{148}}{2}$$

$$X = \frac{14}{2} + \frac{2\sqrt{37}}{2}$$

148
2 ^ 74
2 ^ 37

$\sqrt{148} = \sqrt{4 \cdot 37} = 2\sqrt{37}$

$X = \boxed{7 + \sqrt{37} \text{ hrs apprentice}}$

$X - 2 = 7 + \sqrt{37} - 2$

$= \boxed{5 + \sqrt{37} \text{ hrs experienced}}$

If instructions said to "round to nearest tenth of an hour";

apprentice = 13.1 hrs
experienced = 11.1 hrs

Math 70

(2)

Beach and Fargo are 400 miles apart. A salesperson travels from Beach to Fargo on Monday, and back to Beach on Tuesday. On Tuesday, she drives 10 mph faster. The total time spent driving was $14\frac{2}{3}$ hours. Find her speed in each direction.

$D = R \cdot T$

Mon	400	R	$\frac{400}{R}$
Tues	400	$R+10$	$\frac{400}{R+10}$

$\frac{D}{R} = \frac{R \cdot T}{R}$ Solve for T

$\frac{D}{R} = T$

Total time = time on Monday + Time on Tuesday

$14\frac{2}{3} = \frac{400}{R} + \frac{400}{R+10}$

convert to improper $14 \times 3 + 2 = 44$ numerator (same denom)

$\frac{44}{3} = \frac{400}{R} + \frac{400}{R+10}$

$LCD = 3R(R+10)$

$\frac{44}{3} \cdot 3R(R+10) = \frac{400}{R} \cdot 3R(R+10) + \frac{400}{R+10} \cdot 3R(R+10)$

$\frac{44R(R+10)}{4} = \frac{1200(R+10)}{4} + \frac{1200R}{4}$

Quick tip: Notice all coefficients are divisible by 4!

$11R(R+10) = 300(R+10) + 300R$

$11R^2 + 110R = 300R + 3000 + 300R$

Notice R^2 quadratic $\Rightarrow$ set = 0.

$11R^2 + 110R = 600R + 3000$

$11R^2 - 490R - 3000 = 0$

$R = \frac{-(-490) \pm \sqrt{(-490)^2 - 4(11)(-3000)}}{2(11)}$

Note:
$b^2 - 4ac$
$(-490)^2 - 4(11)(-3000)$
$372100 = 610^2$ (it does factor)

Math 70

(2) cont

$$R = \frac{490 \pm \sqrt{372100}}{22}$$

$$= \frac{490+610}{22}, \quad \frac{490-610}{22} = \frac{-60}{11}$$

$= \boxed{50 \text{ mph on Monday}}$

↑ negative, so this solution is extraneous.

and $R+10 = \boxed{60 \text{ mph on Tuesday}}$

Seeing the results, the factors must give:

$x = 50$ and $x = \frac{-60}{11}$

$(x-50)$ $\qquad 11x = -60$

$(11x+60)$

so we get

$(x-50)(11x+60)$

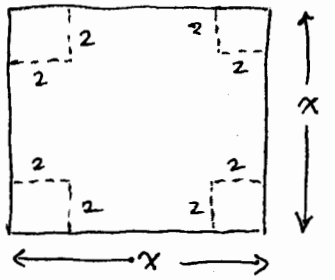
check: $11x^2 + 60x - 550x - 3000$

$11x^2 - 490x - 3000$ ✓

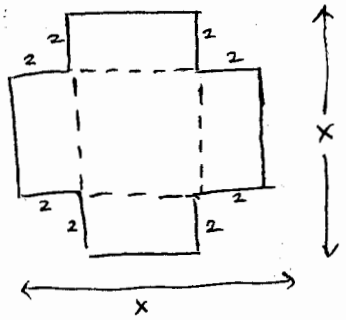
Math 70

(3) **Suppose that an open box is to be made from a square sheet of cardboard by cutting 2-inch-by-2-inch squares out of each corner and folding the sides up. The finished box has volume 128 cubic inches. Findt he original dimensions of the sheet of cardboard.**

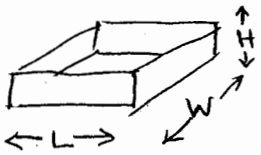
square sheet of cardboard = same dimension for length and width. = x



cut 2-inch squares from each corner
(Do this with a sheet of paper if you're having trouble visualizing it.)



Fold on dotted lines to make a box. (or a tray). It is open on top, with no lid.



$V = L \cdot W \cdot H$

Volume of a rectangular solid

Height is 2", the amount cut away.

Length and Width are both $x-2-2 = x-4$.
(side x, subtract 2 for each missing corner)

Volume is given 128. Substitute:

$$\begin{cases} V = 128 \\ H = 2 \\ L = x-4 \\ W = x-4 \end{cases}$$

$128 = (x-4)(x-4) \cdot 2$

Divide both sides by 2.

$64 = (x-4)(x-4)$

FOIL

$64 = x^2 - 8x + 16$

$0 = x^2 - 8x - 48$

Quadratic formula

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-48)}}{2}$$

$$x = \frac{8 \pm \sqrt{256}}{2}$$

$$x = \frac{8 \pm 16}{2}$$

oops! I could have factored!
$(x-12)(x+4)$

$$x = \frac{24}{2}, \frac{-8}{2}$$

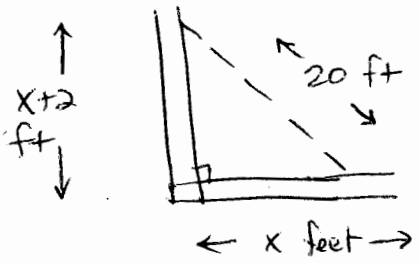
$x = 12, -4$

no lengths are negative.

So original cardboard was **12 inches × 12 inches**

Math 70 — Geometry Example

④ Students cut across the lawn instead of walking on the sidewalks. How many feet do they save by doing so?



The sidewalks form a right angle, so the path across the grass gives a right triangle.

Pythagorean Theorem

$a^2 + b^2 = c^2$

c must be longest side, or hypoteneuse

$x^2 + (x+2)^2 = 20^2$

FOIL

$x^2 + x^2 + 4x + 4 = 400$

collect on LHS, combine like terms

$2x^2 + 4x - 396 = 0$

divide by 2

$x^2 + 2x - 198 = 0$

$x = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-198)}}{2}$

Quadratic formula

$x = \dfrac{-2 \pm \sqrt{4 + 792}}{2}$

$x = \dfrac{-2 \pm \sqrt{796}}{2}$

subtract gives a negative distance, discard this extraneous solution

walk on sidewalks: $x + (x+2) = 2x+2$

Substitute: $2\left(\dfrac{-2+\sqrt{796}}{2}\right) + 2$

$= -2 + \sqrt{796} - 2$

$= \sqrt{796}$

walk on grass $= 20$

Saved $\boxed{\sqrt{796} - 20 \text{ ft}}$ (approx 8.2 ft)

Math 70

(5) A water tank can be filled by the large and small inlet pipes in 3 hours. The large inlet pipe can fill the tank in 2 hours less time than the small inlet pipe. Find the time, to the nearest tenth of an hour, that each pipe takes to fill the tank alone.

$\frac{1}{x-2} + \frac{1}{x} = \frac{1}{3}$

$3x + 3(x-2) = x(x-2)$

$3x + 3x - 6 = x^2 - 2x$

$x^2 - 8x + 6 = 0$

$x = \frac{8 \pm \sqrt{64-4(6)}}{2}$

$x = \frac{8 \pm \sqrt{40}}{2}$

$= \frac{8 \pm 2\sqrt{8}}{2}$

$= 4 \pm 2\sqrt{2} \approx$ 6.8 hr ~~1.2 hr~~

(6) Bill and his son Billy can clean the house together in 4 hours. Billy takes an hour longer by himself than Bill takes by himself. How long, to the nearest hour, does it take each one to clean the house, if working alone?

$\frac{1}{4} = \frac{1}{x+1} + \frac{1}{x}$

$x(x+1) = 4x + 4(x+1)$

$x^2 + x = 4x + 4x + 4$

$x^2 - 7x - 4 = 0$

$x = \frac{7 \pm \sqrt{7^2 - 4(-4)}}{2}$

$= \frac{7 \pm \sqrt{65}}{2} \approx$ 7.53 hrs ~~-.53 hrs~~

(7) At the 2007 Grand Prix of Long Beach auto race, Simon Pagenaud posted the fastest lap speed, but Sebastian Bourdais won the race. One lap through the streets of Long Beach is 10,391 feet (1.968 miles) long. Pagenaud's fastest lap speed was 0.55 foot per second faster than Bourdais's fastest lap speed. Traveling at these fastest speeds, Bourdais would have taken 0.25 second longer than Pagenaud to complete a lap.

a. Find Sebastian Bourdais's fastest lap speed during the race. Round to two decimal places.
b. Find Simon Pagenaud's fastest lap speed during the race. Round to two decimal places.
c. Convert each speed to miles per hour. Round to one decimal place.

$D = R \cdot T$

S.P.	10,391	$x+.55$	$\frac{10391}{x+.55}$
S.B.	10,391	x	$\frac{10391}{x}$
		diff	.25

a) $\frac{10391}{x+.55} = \frac{10391}{x} - .25$

$10391x = 10391(x+.55) - .25x(x+.55)$

$10391x = 10391x + 5715.05 - .25x^2 - .1375x$

$\frac{0}{-.25} = \frac{-.25x^2}{-.25} - \frac{.1375x}{-.25} + \frac{5715.05}{-.25}$

$0 = x^2 + .55x - 22860.2$

$x = \frac{-.55 \pm \sqrt{(.55)^2 - 4(-22860.2)}}{2} = \frac{-.55 \pm \sqrt{91441.1025}}{2}$

$x = 150.92$ ft/sec (discard (−))

$x + .55 = 151.47$ ft/sec

$\frac{\text{ft}}{\text{sec}} \cdot \frac{1\text{ mi}}{5280\text{ ft}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} = \frac{3600}{5280}$

c) $150.92\left(\frac{3600}{5280}\right) =$ 102.9 mph

$151.47\left(\frac{3600}{5280}\right) =$ 103.3 mph Pagenaud

Math 70

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(8) A family drives 500 miles to the Grand Canyon. The return trip was made at a speed 10 mph faster. The total traveling time was $18\frac{1}{3}$ hours. Find the speed to the Grand Canyon and the return speed.

D	R	T
500	x	$\frac{500}{x}$
500	x+10	$\frac{500}{x+10}$

$$\frac{500}{x} + \frac{500}{x+10} = \frac{55}{3}$$

$$LCD = 3x(x+10)$$

$$3 \cdot 500(x+10) + 3x \cdot 500 = 55(x)(x+10)$$

$$1500x + 15000 + 1500x = 55x^2 + 550x$$

$$\frac{0}{5} = \frac{55x^2}{5} - \frac{2450x}{5} - \frac{15000}{5}$$

$$0 = 11x^2 - 490x - 3000$$

$$x = \frac{490 \pm \sqrt{(-490)^2 - 4(11)(-3000)}}{2(11)}$$

$$x = \frac{490 \pm \sqrt{372100}}{22}$$

$$x = \frac{490 \pm 610}{22}$$

$$x = 50 \text{ or } \frac{-60}{11}$$

50 mph going to the Grand Canyon
60 mph returning

Math 70 EXAM #3 Topics List

Math 70-33: Exam #2 Monday, 9 April, 1:20 PM – 2:25 PM
Math 70-34: Exam #2 Thursday, 5 April, 1:20 PM – 2:25 PM

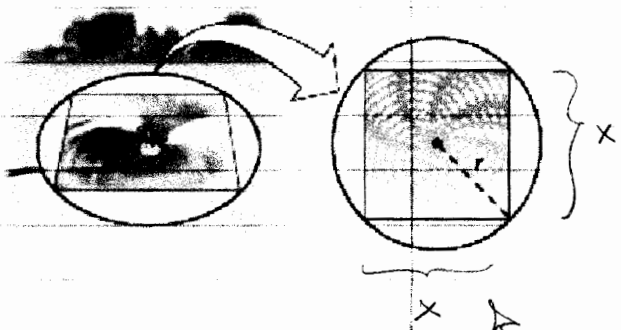
Optional Exam 2 Review session / office hours for any Math 70 student in either section:
Friday, March 23, 1-3 PM, room 342.

Martin-Gay		Bittinger
Lecture sections	Lesson number	Lecture sections
6.1 & 6.2	21	7.1, 7.2 & 7.3
6.2 & 6.3-1st	22	7.4 & 7.5-1st*
6.3-2nd & 6.5	23	7.5-2nd & 7.6
6.6	24	7.7
	25	No lesson (day of Exam 2)
6.7	26	7.8
8.4	27	8.4*
8.1	28	11.1
8.2	29	11.2 & 11.3
apps Q	30	~~11.4~~ 11.5
8.3	31	11.5* 11.4
	32	
	33	
	34	
	35	EXAM 3 (c7, c8, c11.1-11.5)

If you are repeating Math 70 or study with students in other sections of Math 70, it may be helpful to know how the other textbook, Martin-Gay, aligns with your textbook, Bittinger.

Extra

MG 8.3.71 **A sprinkler that sprays water in a circular motion is to be used to water a square garden. If the area of the garden is 720 square feet, find the smallest whole number radius that the sprinkler can be adjusted to so that the entire garden is watered.**



The smallest whole number radius is ☐ feet.

This is an A+ question! You have to remember some geometry. They tell us the area of the square, but that relates to x, the dimension of the square.

Step 1: Find x

$x^2 = 720$ ← area of square

$x = \pm\sqrt{720}$

negative is extraneous

$x = 12\sqrt{5}$

720 = 72 · 10; 72 = 8 · 9; 8 = 4 · 2; 4 = 2 · 2; 9 = 3 · 3; 10 = 2 · 5

$720 = 2^4 \cdot 3^2 \cdot 5$

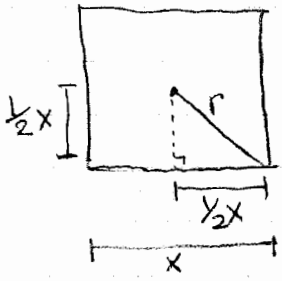
so $\sqrt{720} = 2^2 \cdot 3\sqrt{5}$

$= 12\sqrt{5}$

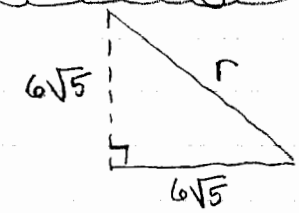
But x is NOT r!

$\frac{1}{2}x \Rightarrow \frac{1}{2}(12\sqrt{5})$

$\frac{1}{2}x \Rightarrow 6\sqrt{5}$



Step 2: Draw a right Δ: Notice that both legs are $\frac{1}{2}x = 6\sqrt{5}$



Step 3: Use Pythagorean Theorem to find r.

$a^2 + b^2 = c^2$

$(6\sqrt{5})^2 + (6\sqrt{5})^2 = r^2$

$36 \cdot 5 + 36 \cdot 5 = r^2$

$72 \cdot 5 = r^2$

$360 = r^2$

$\sqrt{360} = r$

Step 4: calculate r and use next higher whole # for answer.

$r = \sqrt{360} \approx 18.9$

r = 19 feet

Math 70

(21) A holding pen for cattle is square with diagonal 100 m.
a) Find the lengths of the sides.
b) Find the area of the pen.

(22) A window's length is 7.3 inches longer than its width. Its area is 569.9 sq. inches. Find the dimensions.

(23) The base of a triangle is 4 more than twice its height. If the area of the triangle is 42 cm^2, find its base & height.

(24) The number of visitors to U.S. theme parks can be modeled by $V(x) = 0.25x^2 + 2.6x + 315.6$ where $V(x)$ is the number of visitors (in millions) and x is the number of years after 2000.
a) Find the number of visitors is 2005, to nearest million.
b) Project the number of visitors in 2010, to nearest million.

Solve by CTS

(25) $2y^2 - 10y + 11 = 0$ $\quad y = \frac{5}{2} \pm \frac{\sqrt{3}}{2}$

(26) $3x^2 - x - 52 = 0$ $\quad x = -4, \frac{13}{3}$

(27) $9x^2 - 18x = -11$ $\quad x = 1 \pm \frac{i\sqrt{2}}{3}$

(28) $3y^2 - 4y - 1 = 0$ $\quad y = \frac{2}{3} \pm \frac{\sqrt{7}}{3}$

(29) $3x^2 + 2 = 0$ $\quad x = \pm \frac{i\sqrt{6}}{3}$

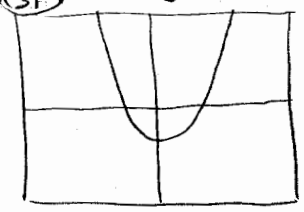
(30) An object thrown upward from the top of a 200ft cliff with initial velocity 12 ft per sec. The height h (from the base of the cliff) after t seconds is

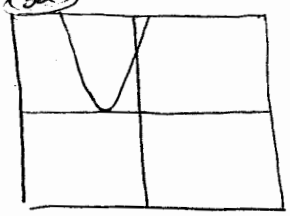
$$h = -16t^2 + 12t + 200$$

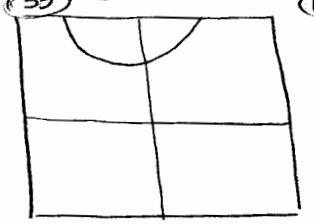
When will it hit the ground? Round to the nearest tenth of a second.

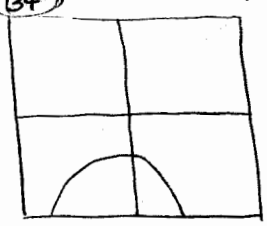
$t = 2.4$ sec

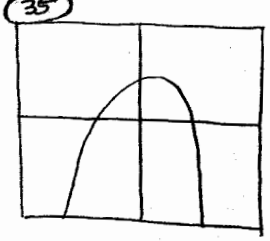
From graph, determine how many solutions and their type.

(31) 
2 real

(32) 
probably 1 real

(33) 
2 complex

(34) 
2 complex

(35) 
2 real

8.3.63 A father and his son can clean the house together in 5 hours. When the son works alone, it takes him an hour longer to clean than it takes his dad alone. Find how long it takes the son to clean alone.

It takes the son ☐ hours to clean alone. (Round to the nearest tenth.)

dad = x hours to do entire job

son = $x+1$ hours

fractions done in 1 hour:

$$\frac{1}{x} + \frac{1}{x+1} = \frac{1}{5}$$

LCD = $5x(x+1)$

$$5(x+1) + 5x = x(x+1)$$

dist

$$5x+5+5x = x^2+x$$

$$10x+5 = x^2+x$$

$$0 = x^2 - 9x - 5$$

QF

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-5)}}{2}$$

$$x = \frac{9 \pm \sqrt{81+20}}{2}$$

$$x = \frac{9 \pm \sqrt{101}}{2}$$

$x \approx 9.5$ or $-.5$ discard negative time

x = dad

Question asks for son.

$9.5 + 1 =$ **10.5 hrs**